Necessary and sufficient conditions for the existence of an image with a given code

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The article introduces an image encoding function which is invariant with respect to affine transform. The properties of the encoding funciton are investigated. Necessary and sufficient conditions are found for a given set of numbers to be a code of nonsingulari image.

Keywords: image code, image encoding, affine equivalence.

Introduction

In pattern recognition in is often required to represent an image consisting of a finite number of points in a form of some code. A possible solution is to simply store coordinates of all points. A drawback here is non-invariance of the code under natural transformations, such as translation, rotation, stretching, etc., and binding of the code to a concrete coordinate system. It is reasonable to consider images mapped on each other by such transformations to be equivalent.

V. N. Kozlov introduced an encoding that is invariant with respect to affine transformations ([4, 5]). It was shown that equality of the codes of two planar images is necessary and sufficient for affine equivalence. In our paper we modify Kozlov's encoding and investigate the properties of the new code. In particular we obtain a criterion that describes all tuples that are codes of some planar images.

Necessary and sufficient conditions for for the existence of a three-dimensional image with two given planar projections were proven in [2].

In [5] V. N. Kozlov proposed the encoding ρ that is invariant with respect to planar invertible affine transformations. Assume that an image consists of n points $a_1, \ldots, a_n$. For every 6-tuple $\{i, j, k, l, m, p\}$ the code equals $\rho_{ijk,lmp} = \frac{S(\triangle a_i a_j a_k)}{S(\triangle a_l a_m a_p)}$, where $S(abc)$ is the area of the triangle abc; 6-tuples are somehow ordered and the corresponding values of ρ form the code. Thus the code of the image $\{a_1, \ldots, a_n\}$ consists of $(C_n^3)^2$ components[2] and is obviously redundant. In our paper we study the degree of this redundancy. We introduce a modified encoding and obtain explicit conditions that a tuple of

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[2] A permutation of vertices does not affect triangle area. So the tuples with permuted (i, j, k) and (k, l, m) are considered as the same tuple.

numbers is an encoding of some image. We extend the result to the case of the original encoding (in an implicit form).

The modified encoding on a 6-tuple $\{i, j, k, k, m, n\}$ equals $r_{ijk,lmp} = \frac{S'(\triangle a_i a_j a_k)}{S'(\triangle a_l a_m a_p)}$, where S' is the oriented area, i.e. the area endowed with the sign that corresponds to the direction of vertex traversal.

The rest of the paper is organized as follows. The basic concepts and notation are introduced in Section 1. The properties of the image code matrix are investigated in Section 2. The main result is formulated and proved in Section 3. Section 4 is a conclusion.

1. Basic notation

Let S' be the oriented triangle area, i.e. $S'(\triangle abc) = S(\triangle abc)$ for the positive triangle orientation and $S'(\triangle abc) = -S(\triangle abc)$ for the negative one. The triangle orientation is considered to be positive if the triangle vertices are traversed in counterclockwise order and negative otherwise.

Suppose that $n \in \mathbb{N}$ and $a_1, \ldots, a_n$ are some points on a plane. The set $A = \{a_1, \ldots, a_n\}$ is said to be an *image*. An image is *degenerate* if all points are collinear; otherwise it is *non-degenerate*. Fix some (Euclidean) coordinate system. The coodrinates of a point a_i are denoted by $X(a_i)$ and $Y(a_i)$. Hereinafter for the sake of convenience individual indices are denoted by lowercase Latin letters.

Multi-indices, i.e. 3-tuples, are denoted by lowercase Greek letters $\alpha, \beta, \gamma, \ldots$ The components of a multi-index are denoted by $\alpha = [\alpha(1), \alpha(2), \alpha(3)]$, and the triangle[1] with the corresponding vertices is denoted by $\triangle_\alpha = \triangle a_{\alpha(1)} a_{\alpha(2)} a_{\alpha(3)}$.

Let us introduce an equivalence relation on the set of multi-indices. Two multi-indices are equivalent (and thus indistinguishable) if one of them can be obtained from the other by a cyclic permutation. More formally, $\alpha \simeq \alpha'$ if and only if the permutation $\begin{pmatrix} \alpha(1) & \alpha(2) & \alpha(3) \\ \alpha'(1) & \alpha'(2) & \alpha'(3) \end{pmatrix} \in S_3$ is even. A multi-index $\bar{\alpha}$ is conjugate to α if the permutation $\begin{pmatrix} \alpha(1) & \alpha(2) & \alpha(3) \\ \bar{\alpha}(1) & \bar{\alpha}(2) & \bar{\alpha}(3) \end{pmatrix}$ is odd. In essence equivalence means that the corresponding triangles have equal areas, whereas conjugation means that the areas of the corresponding triangles differ only in sign. Hereinafter for the sake of brevity the term "multi-index" will be used to identify equivalence classes.

In total, there are C_n^3 different unoriented triangles with vertices from $A = \{a_1, \ldots, a_n\}$, and respectively $N = 2 \cdot C_n^3$ oriented ones, so there are $2C_n^3$ multi-index equivalence classes.

[1] This notation is also used for the case of degenerate triangles, i.e. collinear points.

Enumerate all multi-indices (i.e. the corresponding triangles $\alpha_1, \dots, \alpha_N$). Let $\mathcal{A} = \{\alpha_1, \dots, \alpha_N\}$ be the set of all multi-indices and $E : \alpha_i \mapsto i$ be the enumeration function.

Consider the set of all fractions $r_{ijk,lmp} = \frac{S'(\triangle a_i a_j a_k)}{S'(\triangle a_l a_m a_p)}$. If a triangle $\triangle a_l a_m a_p$ is degenerate, i.e. $S'(\triangle a_l a_m a_p) = 0$, then $r_{ijk,lmp} = \infty$. This set is referred to as the *code* of the image $\{a_1, \dots, a_n\}$. A similar encoding procedure was proposed in [1].

Definition 1. Let $R = (r_{ij})$ be the matrix of the size $N \times N$ such that the element at the intersection of the ith row and jth column is $r_{ij} = r_{\alpha_i,\alpha_j} = \frac{S'(\triangle_\alpha)}{S'(\triangle_\beta)}$. Thus the elements of the code are arranged into a square table with rows and columns enumerated by multi-indices (triangles). The matrix R is the *code matrix* of the image.

Remark 1. Hereinafter we will also use the notation $r_{\alpha\beta} = R_{E(\alpha)E(\beta)}$, i.e. enumerate rows and columns of code matrices by multi-indices.

Example 1. Consider a trapezoid $a_1a_2a_3a_4$ with bases a_1a_2 and a_4a_3 such that $|a_1a_2| : |a_4a_3| = 1 : 2$ (see Fig. 1). Enumerate multi-indices as it is

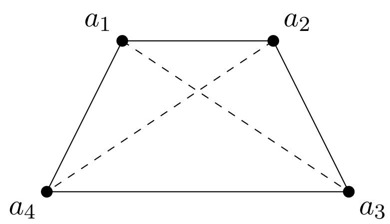

Fig. 1. Example 1.

shown in Table 1. Note that the last four multi-indices are conjugate to the first four, so it is sufficient to construct only the part of the image code matrix corresponding to the first 4 rows and columns. This submatrix is the following: $R_4 = \begin{pmatrix} 1 & 1 & 1/2 & 1/2 \\ 1 & 1 & 1/2 & 1/2 \\ 2 & 2 & 1 & 1 \\ 2 & 2 & 1 & 1 \end{pmatrix}$. The complete code matrix has the following form: $R = \begin{pmatrix} R_4 & -R_4 \\ -R_4 & R_4 \end{pmatrix}$.

Remark 2. Let us enumerate multi-indices in such an order that the second half of the multi-index list is an element-wise conjugation of the first half. More formally $\alpha_{i+N/2} = \overline{\alpha_i}$ for $i = 1, ..., N/2$. Then the code matrix will have the similar form $R = \begin{pmatrix} R' & -R' \\ -R' & R' \end{pmatrix}$.

i	α_i
1	$1, 2, 3$
2	$1, 2, 4$
3	$1, 3, 4$
4	$2, 3, 4$
5	$1, 3, 2$
6	$1, 4, 2$
7	$1, 4, 3$
8	$2, 4, 3$

Table 1. Multi-index enumeration table

2. The properties of code matrices

1) $r_{\alpha\alpha} = 1$ or ∞ for all $\alpha \in \mathcal{A}$ (**reflexivity**).

2) For all $\alpha, \beta \in \mathcal{A}$ such that $r_{\alpha\beta} \notin \{0, \infty\}$ it holds that $r_{\beta\alpha} = r_{\alpha\beta}^{(-1)}$ (**anti-symmetry**[2]).

3) For all $\alpha, \beta, \gamma \in \mathcal{A}$ such that $r_{\alpha\beta}, r_{\beta\gamma} \notin \{0, \infty\}$ it holds that $r_{\alpha\gamma} = r_{\alpha\beta} \cdot r_{\beta\gamma}$ (**transitivity**).

4) Suppose that $\pi, \sigma \in S_3$ and $\alpha, \beta \in \mathcal{A}$. Suppose that $\alpha' = \pi(\alpha)$ and $\beta' = \sigma(\beta)$ are the results of permutations π and σ applied to multi-indices α and β respectively: $\alpha' = [\alpha(\pi(1)), \alpha(\pi(2)), \alpha(\pi(3))]$ and $\beta' = [\beta(\sigma(1)), \beta(\sigma(2)), \beta(\sigma(3))]$. Then either $r_{\alpha'\beta'} = (-1)^\pi \cdot (-1)^\sigma \cdot r_{\alpha\beta}$ or $r_{\alpha\beta} = \infty = r_{\alpha'\beta'}$ (**consistency with index permutations**).

5) Suppose that $i_1, i_2, i_3, i_4 \in \{1, \ldots, N\}$, $\alpha_1 = [i_2, i_3, i_4]$, $\alpha_2 = [i_3, i_4, i_1]$, $\alpha_3 = [i_4, i_1, i_2]$ and $\alpha_4 = [i_1, i_2, i_3]$. Then for any $\beta \in \mathcal{A}$ the equality $r_{\alpha_1\beta} + r_{\alpha_3\beta} = r_{\alpha_2\beta} + r_{\alpha_4\beta}$ holds[3] (**additivity**).

Properties 1–3 are obvious. Property 4 follows from the change in the oriented area sign under permutations of vertices. To prove Property 5 we evaluate the area of the quadrangle $a_{i_1}a_{i_2}a_{i_3}a_{i_4}$ (see. Fig. 2) in two ways:

$$S'(a_{i_1}a_{i_2}a_{i_3}a_{i_4}) = S'(\triangle a_{i_2}a_{i_3}a_{i_4}) + S'(\triangle a_{i_4}a_{i_1}a_{i_2}) = \\ = S'(\triangle a_{i_3}a_{i_4}a_{i_1}) + S'(\triangle a_{i_1}a_{i_2}a_{i_3}).$$

Divide the equality by $S'(\triangle_\beta)$ and obtain Property 5.

It is natural to ask: are these conditions sufficient for an arbitrary matrix to be a code of some image? A counterexample presented below shows that this is not true.

[2]If $r_{\alpha\beta} = 0$ then $r_{\beta\alpha} = \infty$. The converse is generally not true.

[3]If the denominator equals zero, then the equality is treated formally as $\infty + \infty = \infty + \infty$.

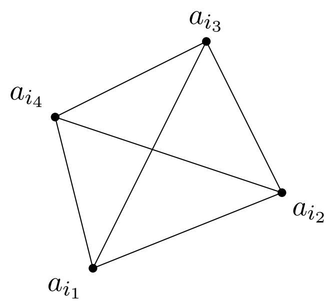

Fig. 2. Area Additivity: $S'_{234} + S'_{124} = S'_{123} + S'_{134}$.

First prove the following auxiliary assertion.

Lemma 1. *Let $\triangle_\beta$ be a non-degenerate triangle, $\rho_\alpha = r_{\alpha\beta}$, $\alpha \in \mathcal{A}$. Fix Euclidean coordinates on a plane. Then there exists an affine transform F such that for any $c_i = F(a_i)$, $i = 1, \dots, n$, it holds that $X(c_i) = \rho_{i,\beta(3),\beta(1)}$, $Y(c_i) = \rho_{i,\beta(1),\beta(2)}$.*

Proof. Without lost of generality assume that $\beta = [1, 2, 3]$. There exists a unique affine transform A such that $a_1 \mapsto c_1(0, 0)$, $a_2 \mapsto c_2(1, 0)$ and $a_3 \mapsto c_3(0, 1)$. Let (x_i, y_i) be the coordinates of $c_i = A(a_i)$, $i = 1, 2, 3$. Then $S'(\triangle c_1c_2c_3) = \frac{1}{2}$, $S'(\triangle c_3c_1c_i) = \frac{1}{2}x_i$ and $S'(\triangle c_1c_2c_i) = \frac{1}{2}y_i$. Thus $\rho_{3,1,i} = \frac{S'(\triangle c_3c_1c_i)}{S'(\triangle c_1c_2c_3)} = x_i$ and $\rho_{3,1,i} = \frac{S'(\triangle c_3c_1c_i)}{S'(\triangle c_1c_2c_3)} = y_i$. Proof is complete. □

Corollary 1. *Let $\triangle_\beta$ be a non-degenerate triangle and*

$$\rho_{i,\beta(1),\beta(2)} = \rho_{j,\beta(1),\beta(2)} = \rho_{k,\beta(1),\beta(2)} = \rho^*,$$

then the points a_i, a_j and a_k are collinear.

Proof. By Lemma 1 there exists an affine transform A such that $A(a_i) = c_i$, $A(a_j) = c_j$,$A(a_j) = c_j$ and $Y(c_i) = Y(c_j) = Y(c_k) = \rho^*$. Hence c_i, c_j and c_k are collinear, therefore a_i, a_j and a_k are collinear as well. □

Corollary 2. *Two non-degenerate images A and B are affine-equivalent if and only if their code matrices are equal for some enumeration of points.*

Remark 3. This corollary is analogous to Theorem 1 from [4] (for a different encoding).

Proof. Necessity is implied by the fact that affine transformations preserve ratios of oriented areas.

Let us prove sufficiency. If an image is non-degenerate, then there obviously exists a non-degenerate triangle $\triangle_\beta(A) = \triangle a_ia_ja_k$. Since the code matrices are equal, the triangle $\triangle_\beta(B) = \triangle b_i b_j b_k$ is also non-degenerate. Consider an image C that consists of the points with coordinates $X(c_i) = \rho_{i,\beta(3),\beta(1)}$, $Y(c_i) = \rho_{i,\beta(1),\beta(2)})$. By Lemma 1 one can construct affine transforms $F_1 : A \to C$ and $F_2 : B \to C$. Thus A and B are affine equivalent. Proof is complete. □

Consider an example that shows that properties 1–5 are not sufficient for the existence of an image with the given code matrix.

Example 2. Consider a regular pentagon with vertices $a_1, \dots, a_5$. Denote the intersection points of the diagonals by $b_1, \dots, b_5$ (see Fig. 3). Place the points m_1, m_2 and m_3 inside the triangles $\triangle a_1 a_2 b_4$, $\triangle a_4 a_5 b_2$ and $\triangle a_3 b_1 b_5$, respectively.

Place unit masses at these points. For the triangle $\triangle a_i a_j a_k$ evaluate the total mass of points located inside it. Endow this mass with the $+/-$ sign depending on the direction of the bypass; the result will be referred to as the pseudo-area of the triangle $\triangle a_i a_j a_k$ and denoted by $S^*(\triangle a_i a_j a_k)$. Obviously pseudo-areas satisfy additivity property. Consider the matrix $R = (r_{\alpha\beta})$, $r_{\alpha\beta} = S^*(\triangle_\alpha)/S^*(\triangle_\beta)$. Properties 1–3 hold for this matrix by construction; Property 4 follows from the definition of pseudo-area; Property 5 is is implied by pseudo-area additivity.

Assume that R is a code matrix for some image $a'_1, \dots, a'_5$. Note that

$$r_{123,123} = r_{124,123} = r_{125,123} = 1,$$

hence by Corollary 1 it holds that the points a'_3, a'_4 and a'_5 are collinear.

Thus the triangle $\triangle a'_3 a'_4 a'_5$ has zero area, thus $r_{345,123} = 0$. But it contradicts to the fact that $r_{345,123} = 1 \neq 0$.

Remark 4. The notion of pseudo-area can be defined more strictly. To do this, map Fig. 3 on the complex plane and interpret points as elements of $\mathbb{C}$. Consider the meromorphic function $f(z) = \frac{1}{z-m_1} + \frac{1}{z-m_2} + \frac{1}{z-m_3}$. Pseudo-area of the triangle $\triangle a_i a_j a_k$ is the contour integral

$$S^*(\triangle a_i a_j a_k) = \frac{1}{2\pi i} \oint\limits_{\triangle a_i a_j a_k} f(z)\mathrm{dz}.$$

3. Main results

We have shown that conditions 1–5 are not sufficient for the existence of an image with the given code matrix. The theorem below answers the question what additional conditions can ensure the existence of such an image.

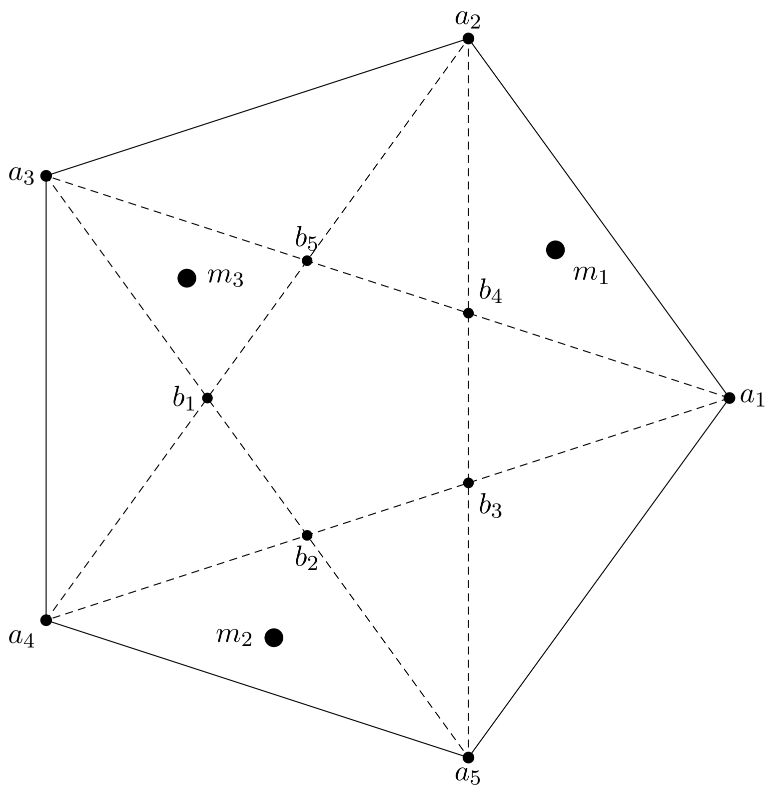

Fig. 3. Example 2.

Theorem 1. *Let the matrix R satisfy conditions 1–5. Suppose that there exist $\alpha, \beta \in \mathcal{A}$ such that $r_{\alpha,\beta} \neq \infty$. Then R is a code matrix of some non-degenerate image if and only if for any $i, j = 1, \ldots, n$ it holds that*

$$\rho_{\beta(1),i,j} = \rho_{i,\beta(3),\beta(1)} \cdot \rho_{j,\beta(1),\beta(2)} - \rho_{j,\beta(3),\beta(1)} \cdot \rho_{i,\beta(1),\beta(2)}, \qquad (1)$$

(here ρ_α stands for $r_{\alpha,\beta}$).

Proof. Necessity. Without loss of generality assume that $\beta = [1, 2, 3]$. Consider the affine transform A from the proof of Lemma 1. It maps the points a_s, $s = 1, 2, 3$, to $c_1 = (0, 0)$, $c_2 = (1, 0)$ and $c_3 = (0, 1)$ respectively. Obviously $S'(\triangle c_1c_2c_3) = \frac{1}{2}$. The transform maps the points a_i and a_j to c_i and c_j with the coordinates $X(c_i) = \rho_{i,3,1}$, $Y(c_i) = \rho_{i,1,2}$ and $X(c_j) = \rho_{j,3,1}$, $Y(c_j) = \rho_{j,1,2}$ respectively. Then the oriented area of $\triangle c_1c_ic_j$ can be found using a well-known formula:

$$S(\triangle c_1c_ic_j) = \frac{1}{2} det \begin{pmatrix} X(c_i) & Y(c_i) \\ X(c_j) & Y(c_j) \end{pmatrix} = \frac{1}{2}(\rho_{i,3,1} \cdot \rho_{j,1,2} - \rho_{j,3,1} \cdot \rho_{i,1,2}).$$

Divide the equality by $S'(\triangle c_1c_2c_3) = \frac{1}{2}$ and obtain the equality (1).

Sufficiency. Suppose that the matrix R satisfies the equality (1). Consider the set of points $\{a_i : i = 1, \ldots, N\}$ with the coordinates $X(a_i) = \rho_{i,3,1}$, $Y(a_i) = \rho_{i,1,2}$. Construct the code matrix $R^* = (r^*_{\alpha\beta})$ for this image. Later we will show that this matrix is equal to the matrix R.

Denote $\rho^*_\alpha = r^*_{\alpha\beta}$. Suppose that $\alpha = [i, j, k]$ and consider the intersection $P = \{i, j, k\} \cap \{1, 2, 3\}$. P is the set of common indices for α and β.

The following cases are possible.

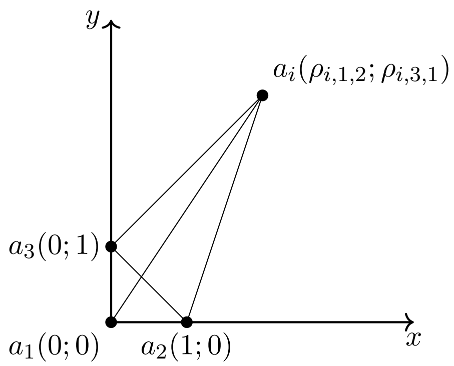

Fig. 4. Case $|P| = 2$.

- The case $|P| = 3$. Then $\alpha = [j, i, k]$ is a permutation of $1, 2, 3$, i.e. either $\alpha = \beta$ or $\alpha = \bar{\beta}$. In this case $S'(\triangle_\beta) = \frac{1}{2}$ and $S'(\triangle_{\bar{\beta}}) = -\frac{1}{2}$. Thus, $\rho^*_\beta = 1 = \rho_\beta$ and $\rho^*_{\bar{\beta}} = 1 = \rho_{\bar{\beta}}$.

- The case $|P| = 2$ and $1 \in P$ (see Fig. 4). In other words, the triangles $\triangle_\alpha$ and $\triangle_\beta$ share two common vertices and one of these vertices is the origin of coordinates. One of the elements i, j, k does not belong to $\{1, 2, 3\}$. Without loss of generality assume that $i \notin \{1, 2, 3\}$. If the remaining indices are 1 and 2, then[4] $\alpha = \gamma$ or $\alpha = \bar{\gamma}$, where $\gamma = [1, 2, i]$. If $\alpha = \gamma$ then

$$S'(\triangle_\gamma) = S'(\triangle a_i a_1 a_2) = \frac{1}{2} Y(a_i) = \frac{1}{2} \rho_{i,1,2}.$$

  Divide the equality by $S'(\triangle_\beta) = 1/2$ and obtain the equality $\rho^*_\gamma = \rho_\gamma$. Otherwise if $\alpha = \bar{\gamma}$, then $\rho^*_\alpha = \rho^*_{\bar{\gamma}} = -\rho^*_\gamma = -\rho_\gamma = \rho_{\bar{\gamma}}$. The case $P = \{1, 3\}$ is considered similarly.

[4]Recall that multi-indices are equivalent with respect to cyclic permutations.

- The case $P = \{2, 3\}$ (see Fig. 4). In other words the triangles $\triangle_\alpha$ and $\triangle_\beta$ share two common vertices a_1 and a_2. One of the elements i, j, k does not belong to the set $\{2, 3\}$. Without loss of generality assume that $i \notin \{2, 3\}$.

  Then either $\alpha = \delta$ or $\alpha = \bar{\delta}$ where $\delta = [i, 3, 2]$. If $\alpha = \delta$ then by Property 5 (additivity)

  $$\rho^*_\alpha = \rho^*_{i,3,2} = \rho^*_{i,3,1} + \rho^*_{i,1,2} - \rho^*_{1,2,3} = \rho_{i,3,1} + \rho_{i,1,2} - \rho_{1,2,3} = \rho_{i,3,2} = \rho_\alpha.$$

  The third equality here follows from the equalities $\rho^*_{i,1,2} = \rho_{i,1,2}$ and $\rho^*_{i,3,1} = \rho_{i,3,1}$ proved above. Otherwise if $\alpha = \bar{\delta}$, then $\rho^*_\alpha = \rho^*_{\bar{\delta}} = -\rho^*_\delta = -\rho_\delta = \rho_{\bar{\delta}}$.

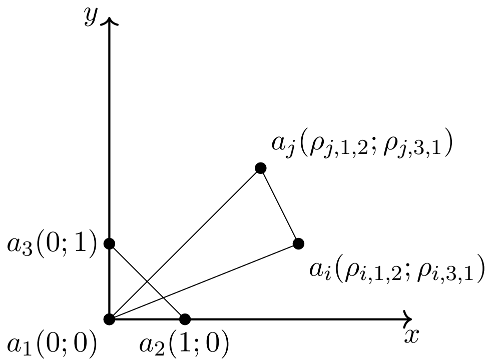

Fig. 5. The case $P = \{1\}$.

- The case $P = \{1\}$ (see Fig. 5). In other words the triangles $\triangle_\alpha$ and $\triangle_\beta$ share a single common vertex located at the origin.

  Without loss of generality assume that the remaining vertices are a_i and a_j. Consider the triangle $\triangle a_1 a_i a_j$ with oriented area

  $$S'(\triangle a_1 a_i a_j) = \frac{1}{2} \cdot \det \begin{pmatrix} X(a_i) & Y(a_i) \\ X(a_j) & Y(a_j) \end{pmatrix} = \frac{1}{2} \cdot (\rho_{i,3,1} \cdot \rho_{j,1,2} - \rho_{j,3,1} \cdot \rho_{i,1,2}).$$

  Divide the equality by $S'(\triangle a_1 a_2 a_2) = 1/2$ and conclude that $\rho^*_{1,i,j} = \rho_{i,3,1} \cdot \rho_{j,1,2} - \rho_{j,3,1} \cdot \rho_{i,1,2} = \rho_{1,i,j}$, where the last equality follows from (1).

- The general case: i, j, k are arbitrary indices. Both ρ and ρ^* are additive. Hence $\rho^*_{i,j,k} = \rho^*_{1,i,j} + \rho^*_{1,j,k} - \rho^*_{1,i,k}$, that (as in the previous case) equals $\rho_{1,i,j} + \rho_{1,j,k} - \rho_{1,i,k} = \rho_{i,j,k}$.

All possible cases are considered, so proof is complete. □

Let us return to the codes that use non-oriented areas ([4]).

Definition 2. A *sign assignment* is an arbitrary set of numbers $s_{\alpha,\beta} \in \{\pm 1\}$, where $\alpha, \beta \in \mathcal{A}$. A sign assignment is *consistent* if for all $\alpha, \beta, \gamma \in \mathcal{A}$ the following equalities hold:

1) $s_{\alpha\beta} \cdot s_{\beta\gamma} = s_{\alpha\gamma}$;

2) $s_{\pi(\alpha)\sigma(\beta)} = (-1)^{\pi} \cdot (-1)^{\sigma} \cdot s_{\alpha\beta}$, $\pi, \sigma \in S_3$.

Remark 5. A consistent sign assignment obviously satisfies the equalities $s_{\alpha\alpha} = 1$ and $s_{\alpha\beta} = s_{\beta\alpha}$ for all $\alpha, \beta \in \mathcal{A}$.

Corollary 3. *The set of numbers $r^*_{\alpha\beta}$ is a code of a non-degenerate image if and only if there exists a consistent sign assignment $s_{\alpha,\beta}$ such that for $r_{\alpha\beta} = s_{\alpha\beta} \cdot r^*_{\alpha\beta}$ the conditions 1–5 and (1) hold.*

Proof. Necessity. Just set $s_{\alpha,\beta} = 1$ if the triangles $\triangle_\alpha$ and $\triangle_\beta$ have the same orientation. Then apply Theorem 1.

Sufficiency. Construct an image with the code matrix $R = ((r_{\alpha\beta}))$ by Theorem 1. Then take $r^*_{\alpha\beta} = |r_{\alpha\beta}|$. □

4. Conclusion

The main result of this paper completely describes the set of codes of non-degenerate images. The future plans are to build similar conditions for other coding functions, e.g., codes that preserve projective equivalence or codes preserving 3-D affine equivalence.

Author would like to express special thanks to A.V. Galatenko for the great help with English translation of this paper.

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